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AN ASYMPTOTIC ANALYSIS OF A TENSILE CRACK IN CREEPING SOLIDS COUPLED WITH CUMULATIVE DAMAGE—PART I. SMALL DAMAGE REGION AROUND THE CRACK TIP

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Abstract—In a material that undergoes creeping deformation coupled with damage, the stress field around the tip of a stationary Mode I crack is treated in two distinct limiting cases. In Part I of the context, a parameter perturbation method is used to study the tip stress field with small damage. Taking a dimensionless coefficient as a perturbation parameter, which characterizes the concerned damage scale, a series of asymptotic solutions can be obtained with the zeroth order approximation being the HRR type of solution. By the results derived one is able to arrive at the distribution of stress and strain, with the coupling effect of damage being taken into account. In this way one is expected to gain a detailed understanding to how the damage influences the HRR tip field. Discussion to another limiting case, which is concerned with the large damage zone very near the crack tip, will be carried out in Part II (Lu, M., Lee, S. B., Kim, J. Y. and May, H. C. (1996). An asymptotic analysis of a tensile crack in creeping solids coupled with cumulative damage—part II. Large damage region very near the crack tip. *Int. J. Solids Structures* (accepted)). © 1997 Elsevier Science Ltd.

1. INTRODUCTION

Under elevated temperature creep conditions in ductile solids, stress field around a Mode I crack subject to external loading has been investigated by many researchers (see, e.g., Riedel and Rice, 1980; Bassani and McClintock, 1981; Ehlers and Riedel, 1981; Hui and Riedel, 1981, etc.). They have analyzed the problem of which macroscopic loading parameter governs crack growth under creep conditions. These studies are primarily based on non-linear fracture mechanics approaches without explicitly introducing quantities related to micro- or meso-damage concept. On the other hand, creeping behaviors in polycrystalline are frequently involved in some microscopic dynamic processes, say, the movement of grain boundary cavitation (e.g., Lee and Miller, 1995). Macroscopic cracks grow by local failure of the highly strained material near the crack tip due to the initiation and joining of microcavities. With these considerations, some researchers have carried out studies to estimate the effect of damage on the tip field based on different assumptions. Among them, for example, Riedel (1981), Bassani and Vitek (1982) calculated the void accumulative damage near a crack tip by using pre-determined stress distributions while effects of cavitation were ignored. Hutchinson (1983) proposed a constitutive law in which a factor was introduced to account for the influence of micro-mechanical voids. With this constitutive equation the stress and strain rate field at the tip of a stationary plane strain crack can be obtained, revealing the role of cavitation on near-tip behavior. Wu *et al.* (1986) considered the growth of cavities by creep deformation (see Hancock, 1976; Budiansky *et al.*, 1983, for details) and investigated the transient crack growth resulting from grain-boundary cavitation.

According to the continuum damage mechanics theory, starting from another standpoint, the process of microscopic cavitation to macroscopic rupture can be described by some macroscopic damage arguments. For isotropic damage a scalar variable, denoted by ω, for instance, is frequently employed to predict the damage, with $\omega = 0$ indicating the

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undamaged case and $\omega = 1$ labeling the ultimately macroscopic rupture. This damage variable can be incorporated into the corresponding constitutive relations to investigate the damage of the media, either on account of experiment evidences or through some theoretical development (cf., Chaboche, 1988a, b; Lemaitre, 1992). The evolution of damage is described by a kinetics equation. Following the continuum damage concept, crack initiation and growth as well as the relevant stress field can be modeled in a manner differing from that of fracture mechanics. In this regard, numeric approaches (e.g., Saanouni *et al.*, 1989; Bassani and Hawk, 1990; Hall and Hayhurst, 1991; Liu *et al.*, 1994, etc.) are usually utilized, mainly due to the complexity of introducing the non-linear damage description, and only in some specific conditions can analytical approximations be obtained (Maas and Pinera, 1985; Zhao and Zhang, 1995, etc.).

In the present paper, the authors attempt to work out an asymptotic stress analysis to the problem with a stationary crack in creep condition coupled with accumulative damage under tensile loading (Mode I). An isotropic damage model is introduced and the damage variable is directly incorporated into the power-law creep constitutive relationship by virtue of the strain-equivalence principle. The damage evolution law is chosen to be the Kachanov–Rabotnov kinetics equation. Since the coupling problem is too sophisticated to perform for a general analysis, we have to confine our attention to this case in which full yielding condition is satisfied and a small damage region can be defined. It is expected that in the case of small damage the HRR (Hutchinson, 1968; Rice and Rosengren, 1968) or RR (Riedel and Rice, 1980) type solution should be the fundamental one, and the damage has effects on the stress field as a perturbation in that the vanishing of damage leads to pure power-law viscous creep behavior. We can thus implement a regular perturbation procedure to the problem and arrive at a series of asymptotic solutions. The structure of the solutions obtained permits one to understand the role of the damage in this situation and how it influences the tip field from another point of view. Besides, the damage field, as well as strains and stresses, can also be predicted within the region concerned.

2. FUNDAMENTAL EQUATIONS

2.1. *Balance equations*

For a stationary problem, one can use the balance equation described below

$$\sigma_{\alpha\beta,\beta} = 0 \quad (\alpha, \beta = 1, 2, 3), \tag{1}$$

where $\sigma_{\alpha\beta}$ is the stress tensor.

2.2. *Compatibility relations* (*plane condition*)

The compatibility equations under plane condition are

$$\dot{\varepsilon}_{\alpha\beta,\alpha\beta} - \dot{\varepsilon}_{\alpha\alpha,\beta\beta} = 0 \quad (\alpha, \beta = 1, 2), \tag{2}$$

in which $\varepsilon_{\alpha\beta}$ denotes the strain (infinitesimal strain) tensor, and the dot over physical quantities means a time derivative with respect to time while the subscript comma indicates the partial derivative with respect to the corresponding spatial coordinate. Since our following discussion is concerned with the plane stress condition, the compatibility equation will be further simplified. Note that the summation convention is applied throughout, otherwise a specification would be given.

2.3. *Constitutive relation and damage evolution law*

For the elastic-secondary creep materials, without being explicitly associated to damage, the constitutive relation can be described by

$$\dot{\varepsilon} = \frac{1+v}{E}\dot{s}_{ij} + \frac{1-2v}{3E}\dot{\sigma}_{kk}\delta_{ij} + \frac{3}{2}B\bar{\sigma}^{n-1}s_{ij}, \tag{4}$$

where

$$s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3 \quad \text{and} \quad \bar{\sigma} = (\tfrac{3}{2}s_{ij}s_{ij})^{1/2} \tag{5}$$

are the deviatoric stress components and the equivalent stress, respectively, E is Young's modules and v the Poisson ratio, B denotes a temperature-dependent coefficient of material. The damage effect can be incorporated into the constitutive equation by using the strain equivalence principle (see Lemaitre, 1992). Thus, eqn (4) can be transformed into

$$\dot{\varepsilon}_{ij} = \frac{1+v}{E}\dot{\tilde{s}}_{ij} + \frac{1-2v}{3E}\dot{\tilde{\sigma}}_{kk}\delta_{ij} + \frac{3}{2}B\bar{\tilde{\sigma}}^{n-1}\tilde{s}_{ij}, \tag{6}$$

where

$$\tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{1-\omega} \quad \text{and} \quad \tilde{s}_{ij} = \frac{s_{ij}}{1-\omega} \tag{7}$$

are the effective stresses and ω represents the damage argument for isotropic damage. Note that

$$\dot{\tilde{\sigma}}_{ij} = \frac{\dot{\sigma}_{ij}}{1-\omega} - \frac{\sigma_{ij}}{(1-\omega)^2}\dot{\omega} \quad \text{and} \quad \dot{\tilde{s}}_{ij} = \frac{\dot{s}_{ij}}{1-\omega} - \frac{s_{ij}}{(1-\omega)^2}\dot{\omega}. \tag{8}$$

Substitution of (7) and (8) into (6) yields

$$\dot{\varepsilon}_{ij} = \frac{1+v}{E}\frac{1}{1-\omega}\dot{s}_{ij} + \frac{1-2v}{3E(1-\omega)}\dot{\sigma}_{kk}\delta_{ij} + \frac{3}{2}B\frac{1}{(1-\omega)^n}\bar{\sigma}^{n-1}s_{ij} - \left[\frac{1+v}{E}\frac{s_{ij}}{(1-\omega)^2} + \frac{1-2v}{3E(1-\omega)}\frac{\sigma_{kk}}{(1-\omega)^2}\delta_{ij}\right]\dot{\omega}. \tag{9}$$

Under a creep condition, the kinetics evolution law of damage can be described by the Kachanov–Rabotnov equation:

$$\mathrm{d}\omega = \left(\frac{\chi(\sigma)}{A}\right)^{\mu}(1-\omega)^{-k}\,\mathrm{d}t \tag{10}$$

or

$$\omega = 1 - \left\{1-(1+k)\int_0^t\left[\frac{\chi(\sigma)}{A}\right]^{\mu}\mathrm{d}\tau\right\}^{1/(1+k)}. \tag{11}$$

in which

$$\chi(\sigma) = \gamma J_0(\sigma) + \beta J_1(\sigma) + (1-\gamma-\beta)J_2(\sigma), \tag{12}$$

where $J_0 = \sigma_1$ indicates the maximum principal stress, $J_1 = tr\sigma_{ij}$ represents the hydrostatic stress, $J_2 = (\frac{3}{2}s_{ij}s_{ij})^{1/2}$ is the Von Mises equivalent stress, namely, $J_2 = \bar{\sigma}$, γ and β are material coefficients dependent on temperature. For many ductile metals, $\gamma \approx 0$ and β is also rather small (see Hayhurst, 1972).

Regardless of that, the constitutive equation is coupled with damage and a damage evolution has been introduced, the stress potential ψ can still be defined as

$$\sigma_{ij} = -\psi_{,ij} + \psi_{,kk}\delta_{ij} \quad (i,j,k = 1,2). \tag{13}$$

In the present context, since we only focus our attention to the plane stress condition, eqn (2) can be simplified to

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\dot{\varepsilon}_{\theta\theta}) + \frac{1}{r^2}\frac{\partial^2 \dot{\varepsilon}_{rr}}{\partial \theta^2} - \frac{1}{r}\frac{\partial \dot{\varepsilon}_{rr}}{\partial r} - \frac{2}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial \dot{\varepsilon}_{r\theta}}{\partial \theta}\right) = 0, \tag{14}$$

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}, \quad \sigma_{\theta\theta} = \frac{\partial^2 \psi}{\partial r^2}, \quad \sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial \theta}\right), \tag{15}$$

where (r, θ) is designated as a polar coordinate system with $\theta = 0$ directly ahead of the crack and the origin at the crack tip.

Limiting our study to such a case in which elastic effects could be neglected as the creep properties are regarded to be more important in dominating the material behavior, the constitutive eqn (9) is then reduced to

$$\begin{aligned} \dot{\varepsilon}_{rr} &= \frac{3}{2}\frac{B}{(1-\omega)^n}\bar{\sigma}^{n-1}s_{rr}, \\ \dot{\varepsilon}_{\theta\theta} &= \frac{3}{2}\frac{B}{(1-\omega)^n}\bar{\sigma}^{n-1}s_{\theta\theta}, \\ \dot{\varepsilon}_{r\theta} &= \frac{3}{2}\frac{B}{(1-\omega)^n}\bar{\sigma}^{n-1}s_{r\theta}. \end{aligned} \tag{16}$$

The damage is mainly accumulated by the creep stresses.

Strictly, the initial condition is that a load is suddenly applied to the cracked specimen at the time $t = 0$. According to the constitutive eqn (9) the instantaneous response of the material is elastic, i.e., at time $t = 0$, the elastic stress distribution prevails in the cracked body. However, details about elastic effect will not be investigated within this context, since, as stated previously, what we are only concerned with here is the creep effect.

Boundary conditions we prescribed on the traction-free crack faces are $\sigma_{ij}n_i = 0$ (n_i = normal vector on crack face), and at infinity.

3. DIMENSIONLESS EQUATIONS

Experimental studies for plenty of ductile metals show that in the vicinity of a crack tip there exists a region where the damage remains relatively small. The fact implies that within a given loading duration, say T^*, there exists a regime with a characteristic radius $R^* = R^*(T^*)$, within which the damage is increasingly large and finally $\omega \to 1$ as $r \to 0$, beyond which, conversely, the damage gradually decreases. In the regime where $r = R^*(T^*)$ the characteristic damage, denoted by δ, is supposed to be much smaller than unity, i.e., $\omega \sim \delta$ while $\delta \ll 1$. To obtain an asymptotic solution which applies to the latter region, a procedure is utilized here to make the equations have a dimensionless form. Noting that the parameter $C(t)$ is used as a measure of the stress intensity for HRR field, thus, in the case of small damage we use C^* [the limiting value of $C(t)$] as a quantity of the characteristic stress intensity.

Let

$$\bar{t}=\frac{t}{T^*},\quad \bar{R}=\frac{r}{R^*},\quad \bar{\sigma}_{ij}=\sigma_{ij}\left(\frac{C^*}{BI_nR^*}\right)^{-1/(n+1)},$$

$$\bar{\psi}=\frac{\psi}{R^{*2}}\left(\frac{C^*}{BI_nR^*}\right)^{-1/(n+1)},\quad \text{and}\quad \bar{\varepsilon}_{ij}=\varepsilon_{ij}\left[\frac{3}{2}B\left(\frac{C^*}{BI_nR^*}\right)^{n/(n+1)}\right]^{-1}. \tag{17}$$

Substitution of (17) into (14) and (16) gives

$$\frac{1}{\bar{R}}\frac{\partial^2}{\partial\bar{R}^2}(\bar{R}\dot{\bar{\varepsilon}}_{\theta\theta})+\frac{1}{\bar{R}^2}\frac{\partial^2\dot{\bar{\varepsilon}}_{rr}}{\partial\theta^2}-\frac{1}{\bar{R}}\frac{\partial\dot{\bar{\varepsilon}}_{rr}}{\partial\bar{R}}-\frac{2}{\bar{R}^2}\frac{\partial}{\partial\bar{R}}\left(\bar{R}\frac{\partial\dot{\bar{\varepsilon}}_{r\theta}}{\partial\theta}\right)=0, \tag{18}$$

$$\begin{aligned}\dot{\bar{\varepsilon}}_{rr}&=\bar{s}_{rr}\bar{\sigma}^{n-1}/(1-\omega)^n,\\ \dot{\bar{\varepsilon}}_{\theta\theta}&=\bar{s}_{\theta\theta}\bar{\sigma}^{n-1}/(1-\omega)^n,\\ \dot{\bar{\varepsilon}}_{r\theta}&=\bar{s}_{r\theta}\bar{\sigma}^{n-1}/(1-\omega)^n\end{aligned} \tag{19}$$

and

$$\omega=1-\left\{1-(k+1)\,\delta\int_0^{\bar{t}}\bar{\chi}^{\mu}(\bar{\sigma})\,\mathrm{d}\bar{\tau}\right\}^{1/(k+1)}, \tag{20}$$

respectively. Here,

$$\delta=T^*\left[\frac{1}{A}\left(\frac{C^*}{BI_nR^*}\right)^{1/(n+1)}\right]^{\mu}, \tag{21}$$

in which I_n is an n-dependent parameter introduced in the literature (Hutchinson, 1968). Evidently, $[C^*/BI_nR^*]^{1/(n+1)}$ represents the characteristic stress around the crack tip, and δ is of an average characteristic damage scale. In addition, R^* dominates a region beyond which all the quantities appeared in the above equations possess the order of unity. Thus, for small damage $\delta \ll 1$ ($R^* \gg 1$), a regular perturbation procedure can be executed to eqns (18)–(20).

4. PERTURBATION EXPANSION AND SOLUTION OF ASYMPTOTIC EQUATIONS

Since in the following discussion only the dimensionless quantities are involved, for convenience and without leading to confusion we rewrite the barred quantities into unbarred ones, but one should have it in mind that they are still dimensionless.

Let

$$\psi=\psi^{(0)}+\delta\psi^{(1)}+\delta^2\psi^{(2)}+\cdots=\sum_{m=0}^{\infty}\delta^m\psi^{(m)}, \tag{22}$$

$$\omega=\omega^{(0)}+\delta\omega^{(1)}+\delta^2\omega^{(2)}+\cdots=\sum_{m=0}^{\infty}\delta^m\omega^{(m)}. \tag{23}$$

Consequently,

$$\sigma_{ij} = \sum_{m=0}^{\infty} \delta^m \sigma_{ij}^{(m)} = -\sum_{m=0}^{\infty} \delta^m \psi_{,ij}^{(m)} + \sum_{m=0}^{\infty} \delta^m \psi_{,kk}^{(m)} \delta_{ij} \quad (i,j,k = r,\theta), \tag{24}$$

in which

$$\begin{aligned}
\sigma_{rr}^{(m)} &= \frac{1}{R}\frac{\partial \psi^{(m)}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \psi^{(m)}}{\partial \theta^2}, \\
\sigma_{\theta\theta}^{(m)} &= \frac{\partial^2 \psi^{(m)}}{\partial R^2}, \\
\sigma_{r\theta}^{(m)} &= -\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi^{(m)}}{\partial \theta}\right)
\end{aligned} \tag{25}$$

and

$$s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3 = \sum_{m=0}^{\infty} \delta^m s_{ij}^{(m)} \quad (i,j,k = r,\theta). \tag{26}$$

Here,

$$\begin{aligned}
s_{rr}^{(m)} &= \frac{1}{R}\frac{\partial \psi^{(m)}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \psi^{(m)}}{\partial \theta^2} - \frac{1}{3}\left(\frac{1}{R}\frac{\partial \psi^{(m)}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \psi^{(m)}}{\partial \theta^2} + \frac{\partial^2 \psi^{(m)}}{\partial R^2}\right), \\
s_{\theta\theta}^{(m)} &= \frac{\partial^2 \psi^{(m)}}{\partial R^2} - \frac{1}{3}\left(\frac{1}{R}\frac{\partial \psi^{(m)}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \psi^{(m)}}{\partial \theta^2} + \frac{\partial^2 \psi^{(m)}}{\partial R^2}\right), \\
s_{r\theta}^{(m)} &= -\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi^{(m)}}{\partial \theta}\right).
\end{aligned} \tag{27}$$

Substitution of eqns (22)–(27) into eqn (18), a family of asymptotic equations can be obtained by means of the usual regular perturbation procedure. Namely, expanding all physical quantities in power series with respect to δ and comparing the terms with the same order of δ, one is then left with the corresponding approximations. For instance, for the δ^0 order solution we have

$$\frac{\partial^2 \dot{\varepsilon}_{\theta\theta}^{(0)}}{\partial R^2} + \frac{2}{R}\frac{\partial \dot{\varepsilon}_{\theta\theta}^{(0)}}{\partial R} - \frac{1}{R}\frac{\partial \dot{\varepsilon}_{rr}^{(0)}}{\partial R} - \frac{2}{R}\frac{\partial^2 \dot{\varepsilon}_{r\theta}^{(0)}}{\partial R \partial \theta} + \frac{1}{R^2}\frac{\partial^2 \dot{\varepsilon}_{rr}^{(0)}}{\partial \theta^2} - \frac{2}{R^2}\frac{\partial \dot{\varepsilon}_{r\theta}^{(0)}}{\partial \theta} = 0, \tag{28}$$

$$\begin{aligned}
\omega &= \omega^{(0)}, \\
\dot{\varepsilon}_{rr}^{(0)} &= s_{rr}^{(0)}[\bar{\sigma}^{(0)}]^{n-1}/(1-\omega^{(0)})^n, \\
\dot{\varepsilon}_{\theta\theta}^{(0)} &= s_{\theta\theta}^{(0)}[\bar{\sigma}^{(0)}]^{n-1}/(1-\omega^{(0)})^n, \\
\dot{\varepsilon}_{r\theta}^{(0)} &= s_{r\theta}^{(0)}[\bar{\sigma}^{(0)}]^{n-1}/(1-\omega^{(0)})^n.
\end{aligned} \tag{29}$$

From eqn (20) one has $\omega^{(0)} \equiv 0$. Then, eqns (25) and (26) offer a description of creep fracture problem for Mode I crack without damage, and the asymptotic solution has been given to it in the works (Hutchinson, 1968; Rice and Rosengren, 1968; Riedel and Rice, 1980). Actually, letting

$$\psi^{(0)}(R,\theta,t) = C(t)R^{s}\Theta^{(0)}(\theta), \quad s = \frac{2n+1}{n+1}, \tag{30}$$

one has

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\theta^2}+\frac{n}{n+1}\right]\left[[\bar{\sigma}_r^{(0)}]^{n-1}\left(2\Theta''^{(0)}+\frac{n+2}{n+1}\frac{2n+1}{n+1}\Theta^{(0)}\right)\right]$$
$$+\frac{n}{(n+1)^2}[\bar{\sigma}_r^{(0)}]^{n-1}\left(\Theta''^{(0)}-\frac{n-1}{n+1}\frac{2n+1}{n+1}\Theta^{(0)}\right)+\frac{6n}{(n+1)^2}([\bar{\sigma}_r^{(0)}]^{n-1}\Theta'^{(0)})' = 0, \tag{31}$$

where

$$\bar{\sigma}_r^{(0)} = \{[\Theta''^{(0)}+s\Theta^{(0)}]^2+[s(s-1)\Theta^{(0)}]^2-s(s-1)[\Theta''^{(0)}+s\Theta^{(0)}]\Theta^{(0)}+3[(s-1)\Theta'^{(0)}]^2\}^{1/2}. \tag{32}$$

The corresponding boundary conditions are

$$\Theta^{(0)}(\pi) = \Theta'^{(0)}(\pi) = 0 \tag{33}$$

and

$$\Theta'^{(0)}(0) = \Theta'''^{(0)}(0) = 0. \tag{34}$$

For the δ^1 order approximation, expanding quantities in (18)–(20) with respect to δ to the first order and comparing the terms with the same order of δ^1, one finds

$$\frac{\partial^2\dot{\varepsilon}_{\theta\theta}^{(1)}}{\partial R^2}+\frac{2}{R}\frac{\partial\dot{\varepsilon}_{\theta\theta}^{(1)}}{\partial R}-\frac{1}{R}\frac{\partial\dot{\varepsilon}_{rr}^{(1)}}{\partial R}-\frac{2}{R}\frac{\partial^2\dot{\varepsilon}_{r\theta}^{(1)}}{\partial R\,\partial\theta}+\frac{1}{R^2}\frac{\partial^2\dot{\varepsilon}_{rr}^{(1)}}{\partial\theta^2}-\frac{2}{R^2}\frac{\partial\dot{\varepsilon}_{r\theta}^{(1)}}{\partial\theta} = 0(R^*\geqslant 1) \tag{35}$$

and

$$\begin{aligned}
\dot{\varepsilon}_{rr}^{(1)} &= s_{rr}^{(1)}[\bar{\sigma}^{(0)}]^{n-1}+(n-1)s_{rr}^{(0)}[\bar{\sigma}^{(0)}]^{n-2}\bar{\sigma}^{(1)}+ns_{rr}^{(0)}[\bar{\sigma}^{(0)}]^{n-1}\omega^{(1)},\\
\dot{\varepsilon}_{\theta\theta}^{(1)} &= s_{\theta\theta}^{(1)}[\bar{\sigma}^{(0)}]^{n-1}+(n-1)s_{\theta\theta}^{(0)}[\bar{\sigma}^{(0)}]^{n-2}\bar{\sigma}^{(1)}+ns_{\theta\theta}^{(0)}[\bar{\sigma}^{(0)}]^{n-1}\omega^{(1)},\\
\dot{\varepsilon}_{r\theta}^{(1)} &= s_{r\theta}^{(1)}[\bar{\sigma}^{(0)}]^{n-1}+(n-1)s_{r\theta}^{(0)}[\bar{\sigma}^{(0)}]^{n-2}\bar{\sigma}^{(1)}+ns_{r\theta}^{(0)}[\bar{\sigma}^{(0)}]^{n-1}\omega^{(1)},
\end{aligned} \tag{36}$$

in which

$$\bar{\sigma}^{(1)} = \tfrac{1}{2}[\bar{\sigma}^{(0)}]^{-1}[2\sigma_{rr}^{(0)}\sigma_{rr}^{(1)}+2\sigma_{\theta\theta}^{(0)}\sigma_{\theta\theta}^{(1)}-\sigma_{rr}^{(0)}\sigma_{\theta\theta}^{(1)}-\sigma_{\theta\theta}^{(0)}\sigma_{rr}^{(1)}+6\sigma_{r\theta}^{(0)}\sigma_{r\theta}^{(1)}], \tag{37}$$

$$\omega^{(1)} = \int_0^t \chi^{\mu}(\sigma^{(0)})\,\mathrm{d}\tau. \tag{38}$$

Note that

$$\bar{\sigma}^{(0)} = C(t)R^{-1/(n+1)}\{[\Theta''^{(0)}+s\Theta^{(0)}]^2+[s(s-1)\Theta^{(0)}]^2-s(s-1)[\Theta''^{(0)}+s\Theta^{(0)}]\Theta^{(0)}$$
$$+3[(1-s)\Theta'^{(0)}]^2\}^{1/2}, \tag{39}$$

$$s_{rr}^{(0)} = \tfrac{1}{3}C(t)R^{-1/(n+1)}[\Theta''^{(0)} + s(s-3)\Theta^{(0)}],$$
$$s_{\theta\theta}^{(0)} = \tfrac{1}{3}C(t)R^{-1/(n+1)}[s(2s-3)\Theta^{(0)} - \Theta''^{(0)}],$$
$$s_{r\theta}^{(0)} = C(t)R^{-1/(n+1)}(1-s)\Theta'^{(0)} \tag{40}$$

and

$$\sigma_{rr}^{(0)} = C(t)R^{-1/(n+1)}[s\Theta^{(0)} + \Theta''^{(0)}],$$
$$\sigma_{\theta\theta}^{(0)} = C(t)s(s-1)R^{-1/(n+1)}\Theta^{(0)},$$
$$\sigma_{r\theta}^{(0)} = C(t)(1-s)R^{-1/(n+1)}\Theta'^{(0)}, \tag{41}$$

$$\omega^{(1)} = \int_0^t \chi^{\mu}(\sigma^{(0)}(\theta))\,\mathrm{d}\tau = A^*(t)R^{-\mu/(n+1)}\chi_T^{\mu}(\theta), \tag{42}$$

where

$$A^*(t) = \int_0^t [C(t)]^{\mu}\,\mathrm{d}\tau. \tag{43}$$

Here, note that $C(t)$ has been non-dimensionalized by C^*, and

$$\chi_T(\theta) = \beta[s^2\Theta^{(0)} + \Theta''^{(0)}] + (1-\beta)\{[\Theta''^{(0)} + s\Theta^{(0)}]^2[s(s-1)\Theta^{(0)}]^2 - s(s-1)[\Theta''^{(0)} + s\Theta^{(0)}]\Theta^{(0)} + 3[(1-s)\Theta'^{(0)}]^2\}^{1/2}. \tag{44}$$

Substituting (36)–(41) into (35), one is left with

$$\frac{\partial^2}{\partial R^2}[R^{-(n-1)/(n+1)}L_{\theta\theta}^{(1)}] + \frac{1}{R}\frac{\partial}{\partial R}[R^{-(n-1)/(n+1)}L_{SR}^{(1)}] - \frac{\partial}{\partial\theta}[R^{-(n-1)/(n+1)-2}L_{R\theta}^{(1)}] + \frac{\partial^2}{\partial\theta^2}[R^{-(n-1)/(n+1)-2}L_{RR}^{(1)}] = \mathscr{J}(R,\theta), \tag{45}$$

where

$$L_{\theta\theta}^{(1)} \equiv f_{\sigma}^{n-1}(\theta)\left[\frac{\partial^2\psi^{(1)}}{\partial R^2} - \frac{1}{3}\Xi^{(1)}\right] + \frac{n-1}{2}f_{\theta\theta}(\theta)f_{\sigma}^{n-3}(\theta)\,\Delta^{(1)}, \tag{46}$$

$$L_{R\theta}^{(1)} \equiv -2f_{\sigma}^{n-1}(\theta)\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi^{(1)}}{\partial\theta}\right) + (n-1)f_{R\theta}(\theta)f_{\sigma}^{n-3}(\theta)\,\Delta^{(1)}, \tag{47}$$

$$L_{RR}^{(1)} \equiv f_{\sigma}^{n-1}(\theta)\left[\frac{1}{R}\frac{\partial\psi^{(1)}}{\partial\theta} + \frac{1}{R^2}\frac{\partial^2\psi^{(1)}}{\partial\theta^2} - \frac{1}{3}\Xi^{(1)}\right] + \frac{(n-1)}{2}f_{RR}(\theta)f_{\sigma}^{n-3}(\theta)\,\Delta^{(1)}, \tag{48}$$

$$L_{SR}^{(1)} \equiv 2L_{\theta\theta}^{(1)} - L_{RR}^{(1)} - \frac{\partial}{\partial\theta}(L_{R\theta}^{(1)}) \tag{49}$$

and

$$\mathscr{J} = \tilde{R}^{(3n-\mu+2)/(n+1)} \cdot n \cdot A^*(t) \left\{ \left(\frac{n+\mu}{n+1}\right)\left(\frac{2n+\mu+1}{n+1}\right)[f_{\theta\theta}(\theta) f_\sigma^{n-1}(\theta)\chi_T^\mu(\theta)] \right.$$
$$- \left(\frac{n+\mu}{n+1}\right)\left[2f_{\theta\theta}(\theta) - f_{RR}(\theta) - 2\frac{\mathrm{d}}{\mathrm{d}\theta} f_{r\theta}\right] f_\sigma^{n-1}(\theta)\chi_T^\mu(\theta)$$
$$\left. + \frac{2(\mu-1)}{n+1}\frac{\mathrm{d}}{\mathrm{d}\theta}[f_{\theta\theta}(\theta) f_\sigma^{n-1}(\theta)\chi_T^\mu(\theta)] + \frac{\mathrm{d}^2}{\mathrm{d}\theta^2}[f_{RR}(\theta) f_\sigma^{n-1}(\theta)\chi_T^\mu(\theta)] \right\}, \tag{50}$$

in which

$$\Xi^{(1)} = \frac{\partial^2 \psi^{(1)}}{\partial R^2} + \frac{1}{R}\frac{\partial \psi^{(1)}}{\partial R} + \frac{1}{R^2}\frac{\partial \psi^{(1)}}{\partial \theta}, \tag{51}$$

$$\Delta^{(1)} = \left\{ [2g_{RR}(\theta) - g_{\theta\theta}(\theta)]\left(\frac{1}{R}\frac{\partial \psi^{(1)}}{\partial R} + \frac{1}{R^2}\frac{\partial \psi^{(1)}}{\partial \theta}\right) + [2g_{\theta\theta}(\theta) - g_{RR}(\theta)]\frac{\partial^2 \psi^{(1)}}{\partial R^2} \right.$$
$$\left. + 6g_{R\theta}(\theta)\left[\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi^{(1)}}{\partial \theta}\right)\right]\right\} \tag{52}$$

and

$$f_\sigma(\theta) = \{[\Theta''^{(0)} + s\Theta^{(0)}]^2 + [s(s-1)\Theta^{(0)}]^2 - s(s-1)[\Theta''^{(0)} + s\Theta^{(0)}]\Theta^{(0)} + 3[(1-s)\Theta'^{(0)}]^2\}^{1/2}, \tag{53}$$

$$f_{\theta\theta}(\theta) = \tfrac{1}{3}[s(2s-3)\Theta^{(0)} - \Theta''^{(0)}], \tag{54}$$

while

$$f_{RR}(\theta) = \tfrac{1}{3}[2\Theta''^{(0)} + s(3-s)\Theta^{(0)}], \tag{55}$$

$$f_{R\theta}(\theta) = (1-s)\Theta'^{(0)}, \tag{56}$$

$$g_{\theta\theta}(\theta) = s(s-1)\Theta^{(0)}, \tag{57}$$

$$g_{R\theta}(\theta) = (1-s)\Theta'^{(0)}, \tag{58}$$

$$g_{RR}(\theta) = s\Theta^{(0)} + \Theta''^{(0)}. \tag{59}$$

Equation (45) is a linear differential equation with variable coefficients and non-homogeneous term. Let

$$\psi^{(1)}(R,\theta) = B(t) R^\alpha \Theta^{(1)}(\theta), \tag{60}$$

then

$$L_{\theta\theta}[\psi^{(1)}(R,\theta)] = B(t) R^\alpha L_{\theta\theta}[\alpha, \Theta^{(1)}(\theta)], \tag{61}$$

$$L_{SR}[\psi^{(1)}(R,\theta)] = B(t) R^\alpha L_{SR}[\alpha, \Theta^{(1)}(\theta)], \tag{62}$$

$$L_{R\theta}[\psi^{(1)}(R,\theta)] = B(t)R^{\alpha}L_{R\theta}[\alpha,\Theta^{(1)}(\theta)], \tag{63}$$

$$L_{RR}[\psi^{(1)}(R,\theta)] = B(t)R^{\alpha}L_{RR}[\alpha,\Theta^{(1)}(\theta)], \tag{64}$$

$$\mathscr{J}[\psi^{(0)}(R,\theta)] = A^{*}(t)R^{(3n+\mu+2)/(n+1)}\mathscr{J}[\Theta^{(0)}(\theta)]. \tag{65}$$

Equating the terms with the same order of index of R on both sides of eqn (45), we have

$$\left(\alpha-2-\frac{n-1}{n+1}\right)\left(\alpha-3-\frac{n-1}{n+1}\right)L_{\theta\theta}[\Theta^{(1)}(\theta)]+\left(\alpha-2-\frac{n-1}{n+1}\right)L_{SR}[\Theta^{(1)}(\theta)]$$
$$-\frac{\mathrm{d}}{\mathrm{d}\theta}L_{R\theta}[\Theta^{(1)}(\theta)]+\frac{\mathrm{d}^2}{\mathrm{d}\theta^2}L_{RR}[\Theta^{(1)}(\theta)] = \left(\frac{n+\mu}{n+1}\right)\left(\frac{n+\mu}{n+1}+1\right)\mathscr{J}[\Theta^{(0)}(\theta)], \tag{66}$$

where

$$\alpha = 2-\frac{1+\mu}{n+1},\quad B(t) = C(t)\int_0^t [C(t)]^{\mu}\,\mathrm{d}\tau. \tag{67}$$

Equation (66) is the governing equation of the angular stress potential with first order approximation, and it poses a boundary-value equation with the boundary conditions as

$$\Theta'^{(1)}(0) = 0,\quad \Theta'''^{(1)}(0) = 0,$$
$$\Theta^{(1)}(\pi) = 0,\quad \Theta'^{(1)}(\pi) = 0. \tag{68}$$

Following the same procedure illustrated above, higher order approximations can be arrived at. Thus, the required asymptotic solution can be described in the way

$$\psi(R,\theta,t) = R^{(2n+1)/(n+1)}C(t)\Theta^{(0)}(\theta)+\delta R^{-(1+\mu)/(n+1)+2}C(t)\int_0^t [C(t)]^{\mu}\,\mathrm{d}\tau\cdot\Theta^{(1)}(\theta)+\cdots \tag{69}$$

and, consequently, we can obtain the stresses distribution. For instance,

$$\sigma_{\theta\theta}(R,\theta,t) = s(s-1)R^{-1/(n+1)}C(t)\Theta^{(0)}(\theta)$$
$$+\delta\alpha(\alpha-1)R^{-(1+\mu)/(n+1)}C(t)\int_0^t [C(\tau)]^{\mu}\,\mathrm{d}\tau\cdot\Theta^{(1)}(\theta)+\cdots. \tag{70}$$

Expansion of eqn (11) to the second order with respect to δ gives

$$\omega(R,\theta,t) = \delta R^{-\mu/(n+1)}C(t)\chi_T^{\mu}[\Theta^{(0)}(\theta)]$$
$$+\delta^2\left(1-\frac{1}{k+1}\right)R^{-2\mu/(n+1)}\int_0^t [C(t)]^{2\mu}\,\mathrm{d}\tau\cdot\chi_T^{2\mu}[\Theta^{(0)}(\theta)]$$
$$-2\mu\delta^2R^{-2\mu/(n+1)}\left[\int_0^t\int_0^{\tau}[C(\tau)]^{\mu}[C(s)]^{\mu}\,\mathrm{d}s\,\mathrm{d}\tau\right]\chi_T^{\mu-1}[\Theta^{(0)}(\theta)]\cdot\chi_T[\Theta^{(1)}(\theta)]+h.o.t., \tag{71}$$

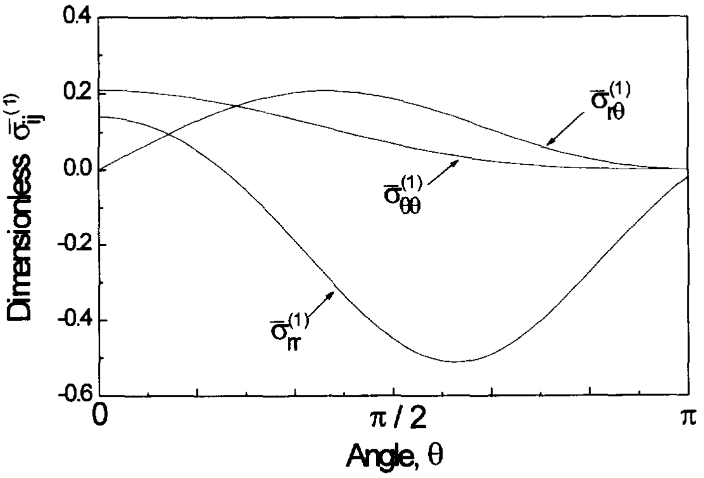

Fig. 1. The angular distribution of the first order perturbation stress field $\bar{\sigma}_{ij}^{(1)}(\theta)$ with $n = 0.8$, $\mu = 1.52$, $k = 2.2$.

where

$$\chi_T[\Theta^{(1)}(\theta)] = \beta tr\sigma_{ij}^{(1)} + \frac{1-\beta}{2\bar{\sigma}^{(0)}}[2\sigma_{rr}^{(0)}\sigma_{rr}^{(1)} + 2\sigma_{\theta\theta}^{(0)}\sigma_{\theta\theta}^{(1)} - \sigma_{rr}^{(0)}\sigma_{\theta\theta}^{(1)} - \sigma_{\theta\theta}^{(0)}\sigma_{rr}^{(1)} + 6\sigma_{r\theta}^{(0)}\sigma_{r\theta}^{(1)}]$$

is a function of θ.

5. RESULTS AND DISCUSSIONS

Computation of eqn (66) with the boundary condition (68) poses a boundary-value problem as the HRR or HR problem does (here, the HRR equation is treated as the δ^0 order approximation). It should be noted, however, that a two-point shooting method has to be utilized, since, unlike the situation in the HRR problem, the boundary condition

$$\Theta^{(1)}(\pi) = \Theta'^{(1)}(\pi) = 0$$

denotes two independent ones. Besides, eqn (66) contains the solution of the HRR problem that must be solved in advance through a boundary-value problem. With these considerations we work out such a numerical procedure to deal with our problem in the following.

As is known, for the HRR problem (the δ^0 order approximation) it can be simply treated in terms of a one-point shooting scheme since, due to the homogeneity of the equation and its boundary condition, one can presume

$$\Theta^{(0)}(0) = 1 \tag{72}$$

through implementing a normalization procedure. The boundary condition (33), as addressed in the literature (Hutchinson, 1968), only yields one independent boundary constraint. In this case a step-advancing bisection technique can be employed to arrive at the corresponding initial value $\Theta''^{(0)}(0)$ and the solution that satisfies the required boundary conditions can be consequently obtained. After solving the δ^0 order approximate solution we may solve the δ^1 order approximation which is defined by a two-point boundary-value problem. For this problem one can utilize the Newton–Raphson approach to obtain the corresponding initial values $\Theta^{(1)}(0)$ and $\Theta''^{(1)}(0)$. In practical computation we find that this two-point shooting problem does not result in more dilemmas in finding the roots, as the corresponding solution curves are quite straightforward and the solution is unique.

Figures 1–4 illustrate the angular distributions of the first order perturbation stress field $\sigma_{ij}^{(1)}(\theta)$ $(i,j = r, \theta)$ when $k = 2.2$, $\mu = 1.52$ with $n = 0.8$, 1.5, 3.1, and 5.5, respectively.

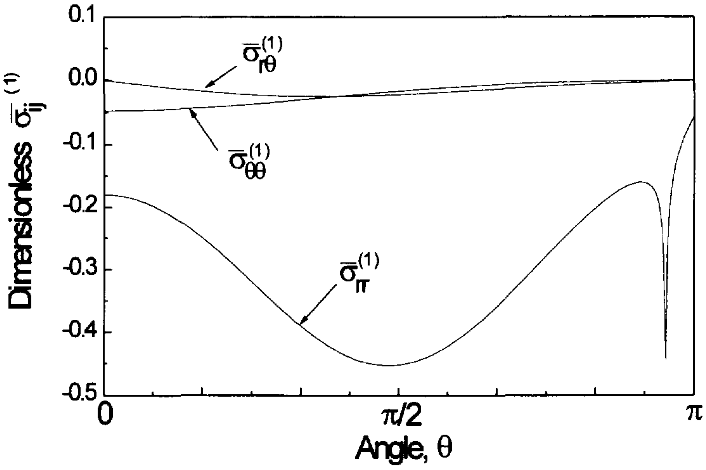

Fig. 2. The angular distribution of the first order perturbation stress field $\bar{\sigma}_{ij}^{(1)}(\theta)$ with $n = 1.5$, $\mu = 1.52$, $k = 2.2$.

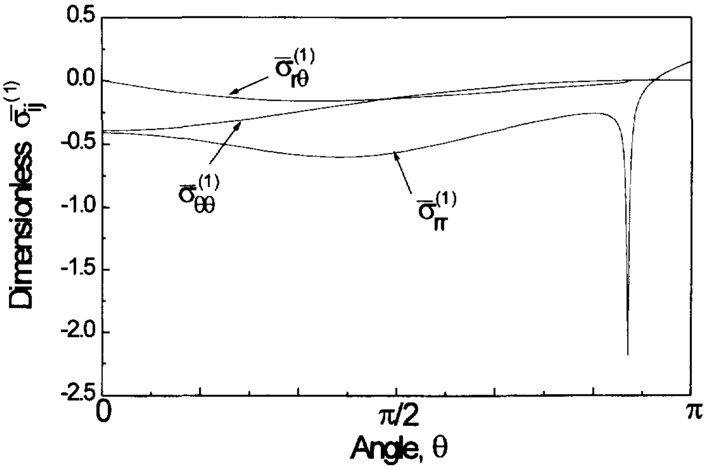

Fig. 3. The angular distribution of the first order perturbation stress field $\bar{\sigma}_{ij}^{(1)}(\theta)$ with $n = 3.1$, $\mu = 1.52$, $k = 2.2$.

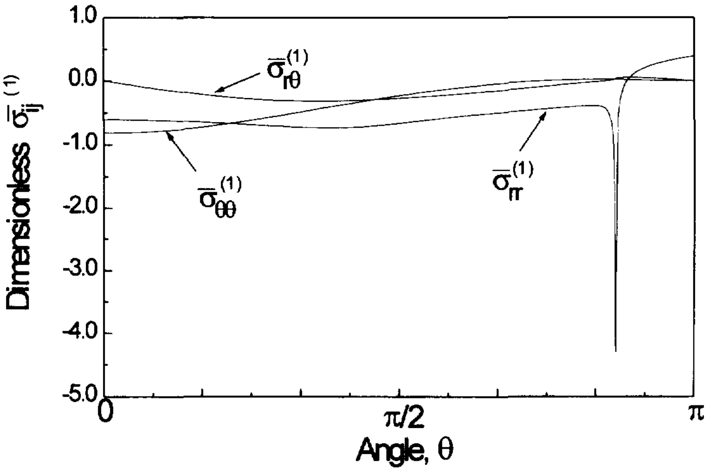

Fig. 4. The angular distribution of the first order perturbation stress field $\bar{\sigma}_{ij}^{(1)}(\theta)$ with $n = 5.5$, $\mu = 1.52$, $k = 2.2$.

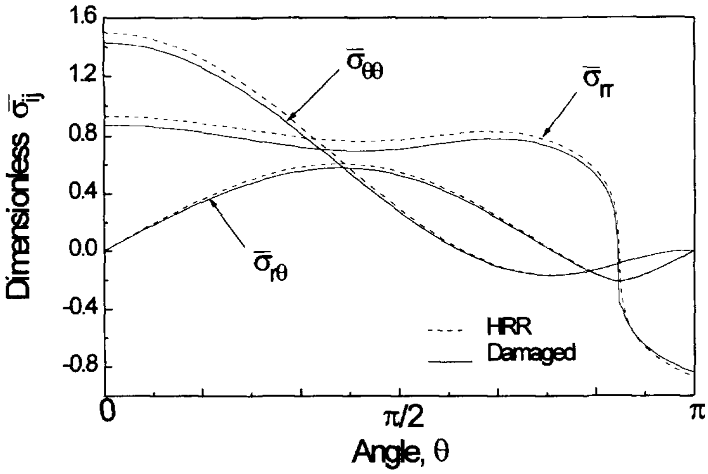

Fig. 5. The angular distribution of dimensionless stresses for HRR field and damaged field with $\delta = 0.1$, while $n = 4.64$, $\mu = 1.47$, $k = 4.94$.

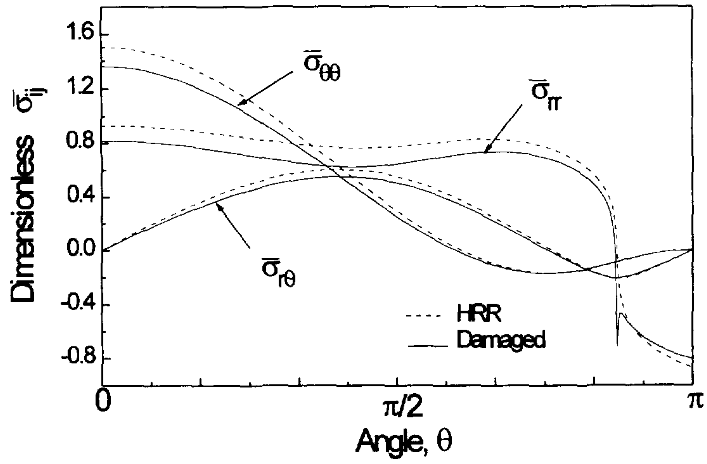

Fig. 6. The angular distribution of dimensionless stresses for HRR field and damaged field with $\delta = 0.2$, while $n = 4.64$, $\mu = 1.47$, $k = 4.94$.

It should be noted that in the computing of eqn (66) the δ^0 order solution $\Theta^{(0)}(\theta)$ as well as $\Theta'^{(0)}(\theta),\ldots,\Theta^{iv(0)}(\theta)$ are used. For $n > 1$ there exist sharp jumps at a certain point in the interval $[0, \pi]$ for $\Theta^{iv(0)}(\theta)$. It can be readily seen that the larger the value of n, the sharper the jump is. However, one needs to pay little attention to it in numerical calculation since the discontinuity only occurs in the non-homogeneous term on the right side and in this case no special numeric treatment is needed. Using the conventional fourth-order Runge–Kutta scheme and $\pi/12{,}000$ integral stepsize, the results obtained are shown to be sufficiently accurate and stable.

Figures 5 and 6 show the angular distribution of dimensionless stresses for HRR field and damaged field with $\delta = 0.1$ and 0.2, respectively. These results are obtained with the first order approximation while $n = 4.64$, $\mu = 1.47$ and $k = 4.94$. The HRR field is obtained by $\delta = 0$. The difference in stresses between undamaged and damaged media increase as δ increases.

Figures 7 and 8 illustrate the result of angular distribution of dimensionless strains with the first order approximation when $\delta = 0$ (HRR field), $\delta = 0.1$ and 0.2.

Figure 9 shows the angular distribution of damage with $\delta = 0.1$ and 0.2 for $n = 4.64$, $\mu = 1.47$ and $k = 4.94$. Since in the majority of the literature the time-dependent function $C(t)$ contains a singularity when $t = 0$, the time integral involved in (67) is not starting from $t = 0$ but commencing at a certain other value, for instance, t_T, the transition time from small yielding to large yielding condition, to the characteristic loading time T^*. This

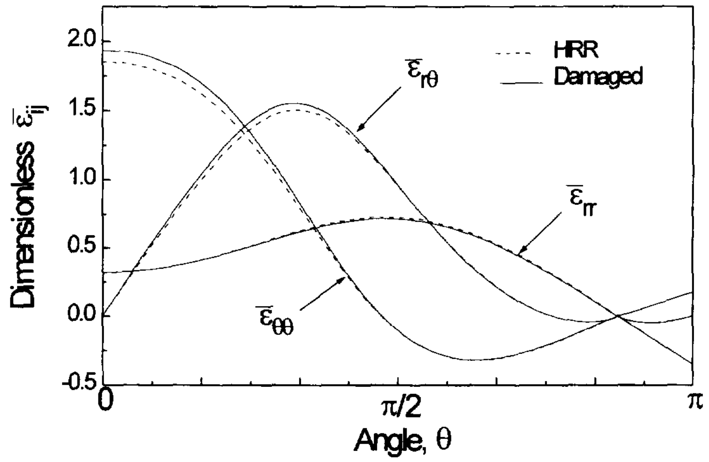

Fig. 7. The angular distribution of dimensionless strains for HRR field and damaged field with $\delta = 0.1$, while $n = 4.64$, $\mu = 1.47$, $k = 4.94$.

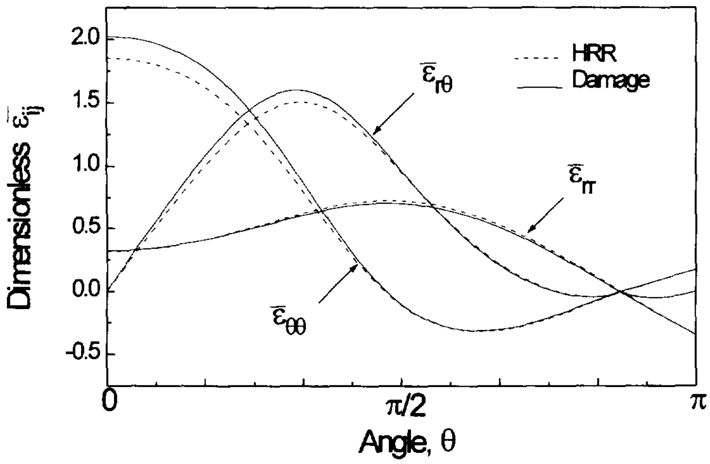

Fig. 8. The angular distribution of dimensionless strains for HRR field and damaged field with $\delta = 0.2$, while $n = 4.64$, $\mu = 1.47$, $k = 4.94$.

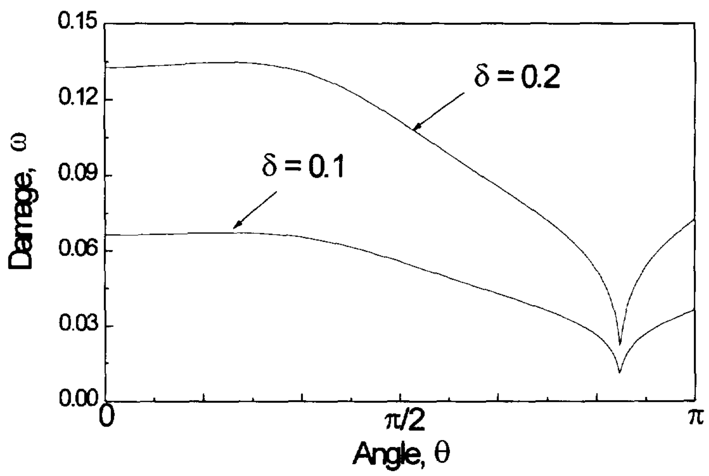

Fig. 9. The angular distribution of damage with $\delta = 0.1, 0.2$, while $n = 4.64$, $\mu = 1.47$, $k = 4.94$.

is acceptable if considering this fact that our analysis and the solution obtained apply to this case in which instantaneous elastic responses in short time are neglected.

The constant C^* is determined by the formulation (see Kumer *et al.*, 1981, for details) and the material constants B, n, k, μ, A are obtained through fitting the experimental data in the paper (Hayhurst, 1972b).

6. CONCLUDING REMARKS

Based on a regular parameter perturbation approach the interaction between creep fracture and damage has been investigated in this paper. From the solutions obtained one can, within the accuracy achieved, realize the distribution of the damage and the effect of the damage on the stress field. In this paper the stress field is provided with the δ^1 order accuracy, however, in terms of the stress distribution the damage distribution with δ^2 approximation can be arrived at. This implies that prediction of damage distribution has higher accuracy. Strain field can also be obtained in a straightforward way by inserting the stresses and damage solved into the corresponding constitutive equations.

It should be noted that although we utilize a parameter perturbation method to deal with our problem, the asymptotic result exhibits a coordinate perturbation form. However, one should not use the result to analyze the singularity since validity of our solution is limited to the case $R^* \geqslant 1$. In another work, Part II of the context, we will discuss the transition behavior from the damage to fracture of the body as $R^* \to 0$.

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